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Topic 08 : Classification1

Part 01 : Overview

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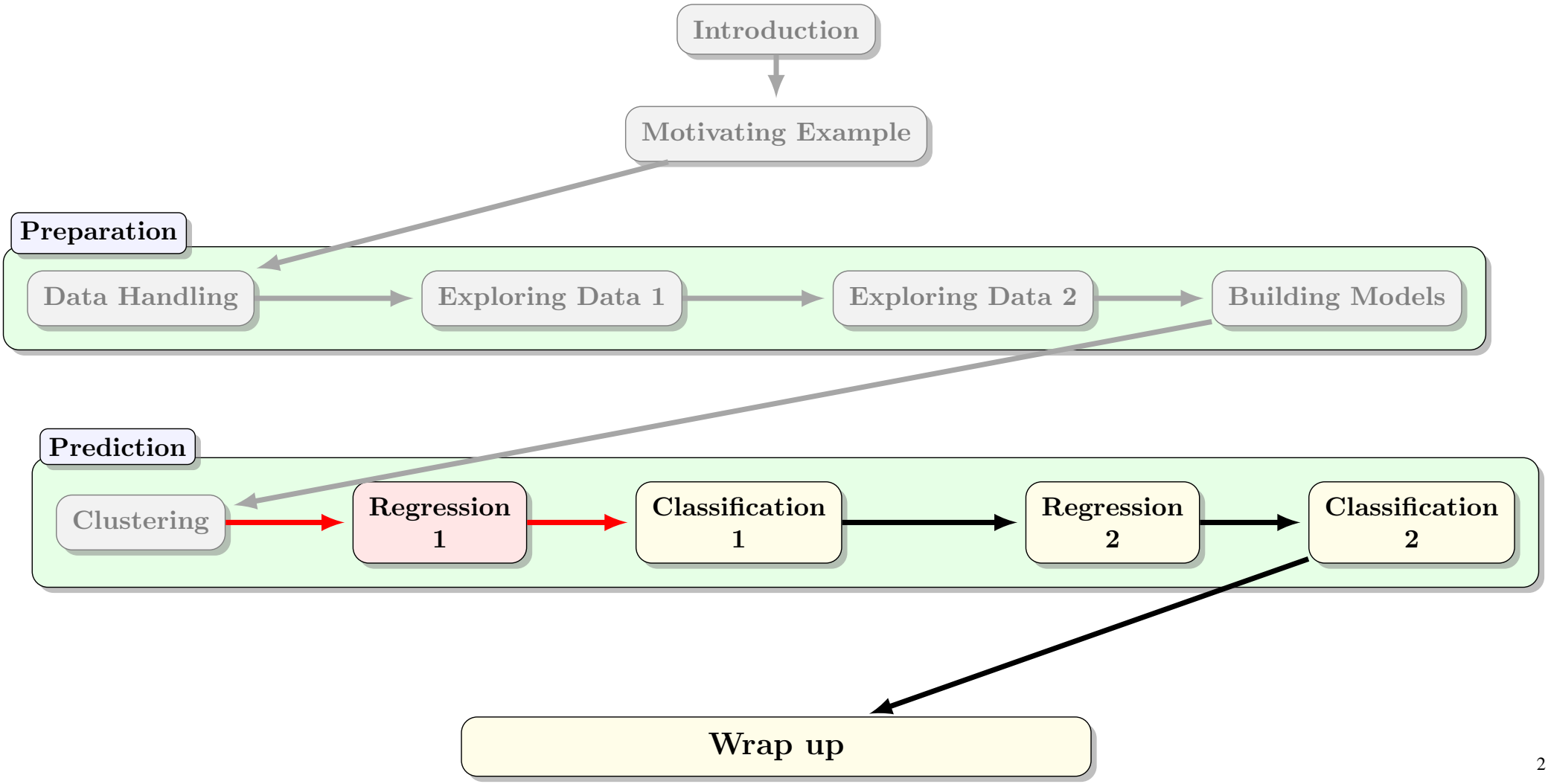
Autumn Semester, 2024

Outline

- How classification differs from regression
- Classification metrics
- Logistic regression in practice - Iris dataset
- Logistic regression - how it works

Data Mining (Week 8)



Acknowledgment

Thanks to Dr Kieran Murphy for some of today's slides.

Introduction to Classification

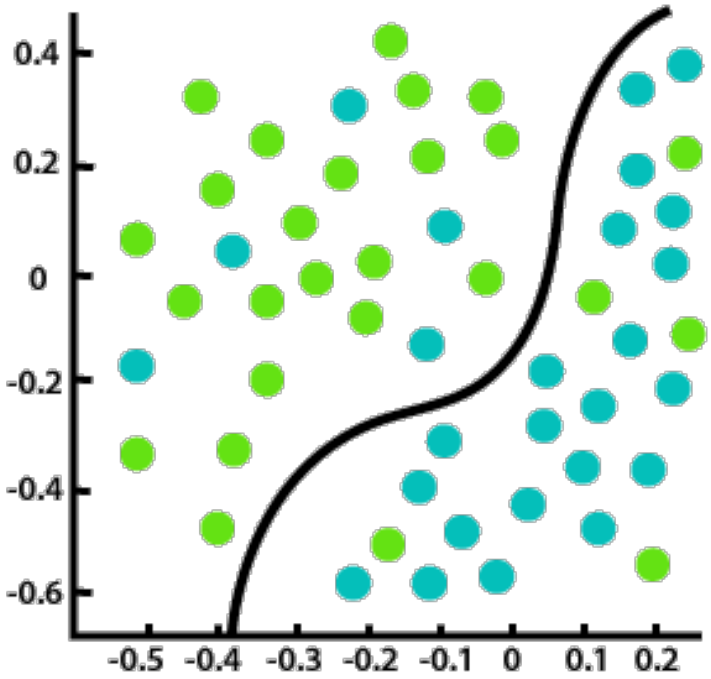
Definition 1 (Classification)

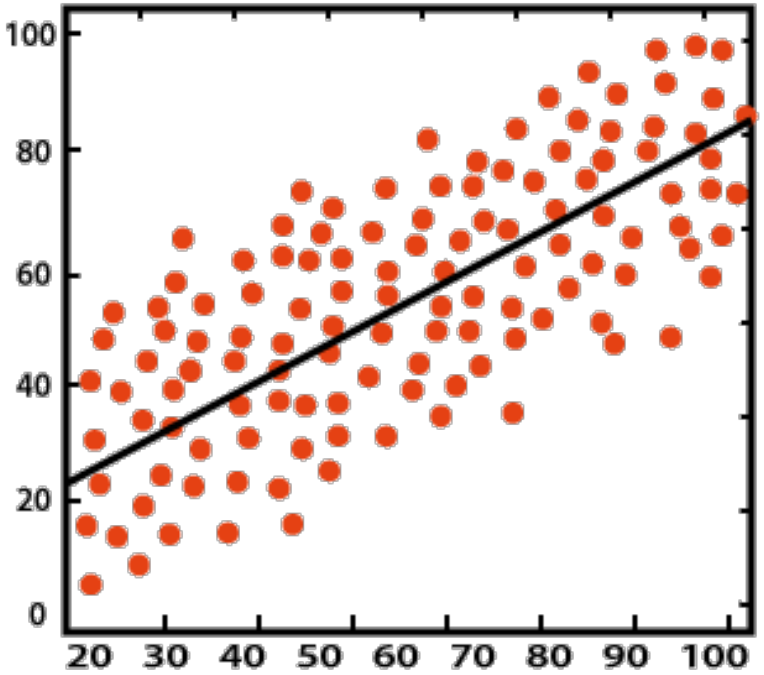
Classification aims to learn a function that takes attribute values and predicts a categorical/qualitative value, such as membership of a class, existence of an effect, etc. The attributes can be categorical or numeric. As with linear regression, classification is an example of supervised learning. It differs from regression because regression predicts a numeric response.

- Some *classifiers* generate class membership probabilities (numeric) en route to predicting class membership (of the most likely class), so the distinction is not always clear-cut.
- There is essentially one regression algorithm (with many variants/enhancements/implementations) but there are *many* classification algorithms.
- You have seen 1 already (KNN) and we introduce another algorithm today.

Classification vs Regression

Supervised data models have a target.
If target is quantitate (continuous) then have a regression model, if categorical then classification model.





Classification models aim to:
- predict class/label for each new observation,
- define a decision boundary between classes,
- and possibly the probability of being in each class.

Regression models aim to
- predict a continuous value for each new observation.

Classification vs Regression

- Unlike regression, statistical distributions play a limited role in evaluating a classifier:
  - Scope for hypothesis testing is limited (there is no equivalent of the `statsmodels` diagnostic output (covered in topic 8).
  - Rely on empirical metrics — accuracy, precision, recall, f1-score, auc, ...
- Classification metrics tend to be easier to use/understand than those in regression — classification metrics are based on counts of correct (or incorrect) cases divided by a subset of cases.
- Central concept in classification model is the confusion matrix:

Actual \ Predicted	Negative	Positive	
Negative	✔ True Negative (*TN*)	Type I error False Positive (*FP*)	*N*
Positive	Type II error False Negative (*FN*)	✔ True Positive (*TP*)	*P*
	$\hat{N}$	$\hat{P}$	*T*

Unbalanced Classification Datasets

Practical classification datasets are often unbalanced — where the frequency of the classes in the target are very uneven:

- Telecommunication customer churn datasets. Churn rate of 2%–10%.
- Credit Card Fraud Detection 0.172% (492 frauds / 284,807 transactions).
- National Institutes of Health Chest X-Ray Dataset 14 cases, (size 13 to 3,044) in 5,606 cases

Solutions

Use suitable metrics and/or

Summary of Classification Models

Model	Data Pre-processing*		Impact from		Summary
	Normalisation	Scaling	Collinearity	Outliers	
KNN	✔	✔	✔	✘	• Local approximation, lazy learner • Heavy computational requirements
Logistic Regression	✔	✘	✔	✔	• Descriptive with good accuracy • Reasonable computational requirements
Naïve Bayes	NA	NA	✔	✘	• Works with categorical features only • Suitable for small train datasets
Decision Tree	✘	✘	✔	✔	• Easy to setup and interpret (XAI). • Robust to missing data but not to noise. • Slow training for larger sets.
Random Forest (Not this module)	✘	✘	✘	✘	• High prediction accuracy • Limited explainability • Works with both continuous and categorical features
Support Vector Classifier (Not this module)	✘	✘	✘/✔	✔	• High prediction accuracy • Explainability depends on kernel • Computational effort depends on kernel
Neural Networks (Not this module)	✘	✔	✔	✔	• High prediction accuracy • Self-extract features • Heavy computational requirements

*Use `StandardScaler`, or `RobustScaler` if have outliers.

Lazy vs Eager Learners

Lazy learner

Stores training data (or only minor processing) and uses this to compute prediction when given test data.

- Does not generalise until after training
- Does not produce a standalone model
- Training data must be kept for prediction
- Local approximations
- Often based on *search*
- If new data is just added to the training data, it can respond more easily to changing conditions

Eager learner

Builds a model from the train set, before receiving new data for prediction

- Training has an extra goal: to generalise from the data
- Training has an extra output: standalone model
- Training data can be discarded after use
- Local and/or global approximations
- Based on *computation*
- Models *drift* with time, so not suited to highly dynamic contexts, as it needs retraining

Usually an (eager) model requires much less memory than a (lazy) training set.

A Non-perfect Test — Type I and Type II Errors

Consider an imperfect test with two outcomes, there are four possible outcomes:

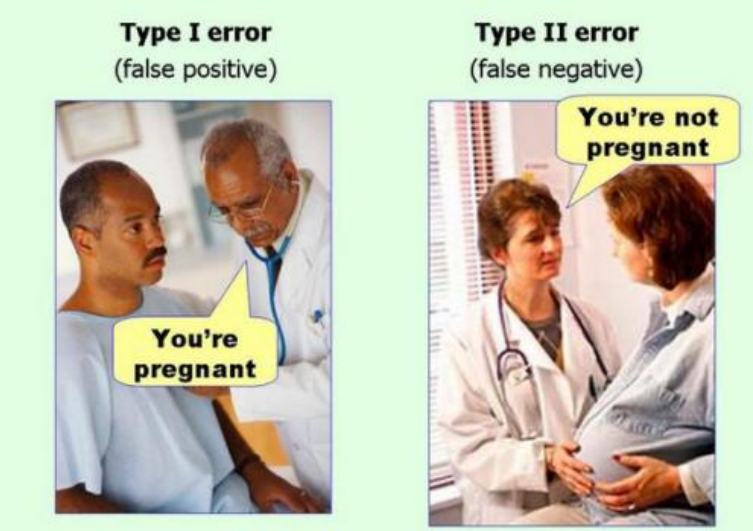
Confusion Matrix

Actual \ Predicted	Negative	Positive	
Negative	✔ True Negative (TN)	Type I error False Positive (FP)	N
Positive	Type II error False Negative (FN)	✔ True Positive (TP)	P
	$\hat{N}$	$\hat{P}$	T

- If the test is applied to $T = P + N = \hat{P} + \hat{N}$ observations / subjects / instances then we have four independent quantities TP, TN, FP, and FN.
- How do we combines these quantities into a single metric ?
- The fraction of correct results seems like a good idea

$$\text{accuracy} = \frac{TP + TN}{P + N}$$

But what happens, if we are testing for an rare event? Maximising accuracy will result in the test always returning negative.



- Ideally we want the probability of either error to be zero but that may not be possible.
- Depending on the conditions we often modify the test to reduce probability of the type of error we don't want at the expense of increasing the probability of the other — think court case vs medical condition.

Confusion matrix (Contingency table) Metrics

Accuracy — how well model is trained and performs in general

$$\textbf{Accuracy} = \frac{\mathrm{TP} + \mathrm{TN}}{\mathrm{P} + \mathrm{N}}$$

(How often is the classifier correct?)

Actual \ Predicted	Negative	Positive	
Negative	✔ True Negative (*TN*)	Type I error False Positive (*FP*)	*N*
Positive	Type II error False Negative (*FN*)	✔ True Positive (*TP*)	*P*
	$\hat{N}$	$\hat{P}$	*T*

- False negative rate (FNR) $= \frac{FN}{\mathrm{P}} = 1 - TPR$
- **Sensitivity** = **Recall** = True positive rate (TPR) $= \frac{TP}{\mathrm{P}} = 1 - FNR$

  (Of positive cases that exist how many did we mark positive?)

Recall — important when the costs of false negatives are high

- **Specificity** $= \frac{\mathrm{TN}}{\mathrm{N}} = 1 - \mathrm{FPR}$

  (When it's actually no, how often does we predict no?)

  (Of cases that are negative, how many did we mark negative?)
- False positive rate (FPR) = false acceptance $= \frac{FP}{\mathrm{N}} = 1 - \text{Specificity}$
- **Precision** = positive predictive value (PPV) $= \frac{TP}{\hat{\mathrm{P}}} = \frac{TP}{\mathrm{TP} + \mathrm{FP}}$

  (Of cases that we marked positive, how many were correct?)

Precision — important when the costs of false positives are high

Confusion matrix (Contingency table) Metrics II

F_1 Score

The F-measure or balanced F-score (F_1 score) is the harmonic mean of precision and recall:

$$F_1 = 2\left[\frac{1}{\frac{1}{\text{precision}} + \frac{1}{\text{recall}}}\right] = 2\left[\frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}}\right]$$

A word of Caution ...

Consider the three binary classifiers A, B and C

	A		B		C	
	T	F	T	F	T	F
T	0.9	0.1	0.8	0	0.78	0
F	0	0	0.1	0.1	0.12	0.1

Metric	**A**	**B**	**C**	(best)
Accuracy	0.9	0.9	0.88	**A,B**
Precision	0.9	1.0	1.0	**B,C**
Recall	1.0	0.888	0.8667	**A**
F-score	0.947	0.941	0.9286	**A**

Clearly classifier *A* is useless since it always predicts label T regardless of the input. Also, B is slightly better than C (lower off- diagonal total). Yet look at the performance metrics – *B is never the clear winner, and A has the best Accuracy, Recall and F_1-score, even though it just predicts the majority class! You can see why imbalanced data causes problems...*

We use some metrics because they are easy to understand, and not because they always give the "correct" result.

Mutual Information is a Better Metric

The mutual information between predicted and actual label (case) is defined

$$I(\hat{y}, y) = \sum_{\hat{y}=\{0,1\}} \sum_{y=\{0,1\}} p(\hat{y}, y) \log \frac{p(\hat{y}, y)}{p(\hat{y})p(y)}$$

where $p(\hat{y}, y)$ is the joint probability distribution function.
This gives the intuitively correct rankings $B > C > A$

Metric	**A**	**B**	**C**
Accuracy	0.9	0.9	0.88
Precision	0.9	1.0	1.0
Recall	1.0	0.888	0.8667
F-score	0.947	0.941	0.9286
Mutual information	**0**	**0.1865**	**0.1735**

Micro Average vs Macro Average Performance

In a multi-class classifier we have more than two classes.

To combine the metrics for individual classes to get an overall system metrics, we can apply either

Micro-Average Method `weighted avg`

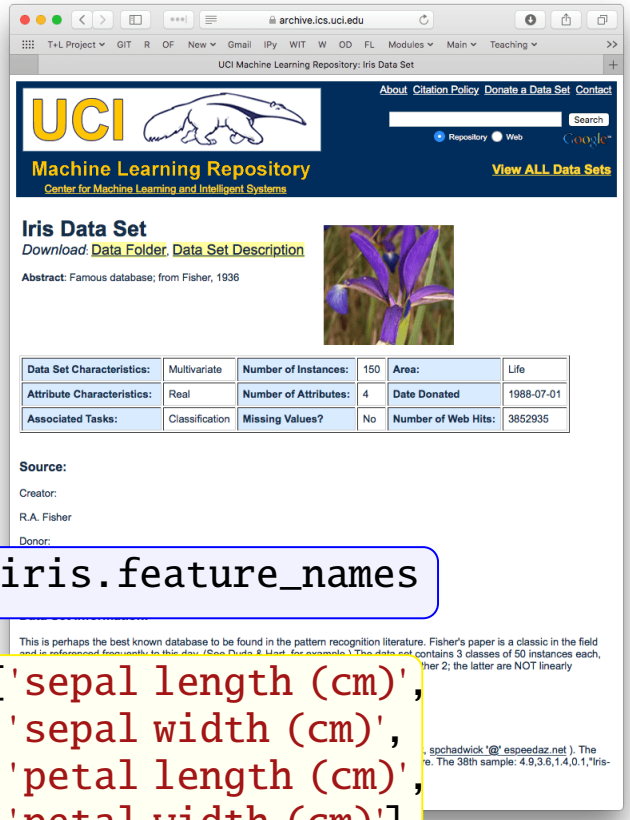
Sum up the individual true positives, false positives, and false negatives of the system for different classes and then apply totals to get the statistics.

Macro-average Method `macro avg`

Average the precision and recall of the system on different classes.

	precision	recall	f1-score	support
setosa	1.00	0.95	0.97	19
versicolor	0.81	0.74	0.77	23
virginica	0.71	0.83	0.77	18
accuracy			0.83	60
macro avg	0.84	0.84	0.84	60
weighted avg	0.84	0.83	0.84	60

Example: IRIS Dataset — Load




```
iris.feature_names
```

```
['sepal length (cm)',
 'sepal width (cm)',
 'petal length (cm)',
 'petal width (cm)']
```

```
from sklearn import datasets
iris = datasets.load_iris()

df = pd.DataFrame(iris.data)
df.columns = iris.feature_names
df['target'] = iris.target_names[iris.target]
df.sample(4)
```

	sepal length (cm)	sepal width (cm)	petal length (cm)	petal width (cm)	target
56	6.3	3.3	4.7	1.6	versicolor
73	6.1	2.8	4.7	1.2	versicolor
49	5.0	3.3	1.4	0.2	setosa
18	5.7	3.8	1.7	0.3	setosa

The data set contains, four numeric features, 3 classes of 50 instances each, where each class refers to a type of iris plant. One class is linearly separable from the other 2; the latter are NOT linearly separable from each other.

Example: IRIS Dataset — Preprocess Data

We will cover some classifiers in a moment, but for now just treat the classifiers (`LogisticRegression`) as a black box and focus on the general process:

Extract the data (features and target)

The IRIS dataset has 4 features, but to simplify visualisation we are only going to use the first two[†] ('sepal length' and 'sepal width'):

```
dataset_name = "IRIS"
X, y, target_names = iris.data[:,:2], iris.target, iris.target_names
```

Split dataset into `train` and `test`

We will keep 40% of the data for testing. Setting the parameter `random_state` to a value means that we will get a random — but still reproducible — split.

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y, train_size=0.6, random_state=666)
```

:

[†] Python for Data Science — Cheat Sheet Numpy Basics

Example: IRIS Dataset — Fit Model and Predict

Select classifier

Scikit-learn supports a large set of classifiers, and aims to have a consistent interface to all. First import classifier and create instance ...

```
from sklearn.linear_model import LogisticRegression
model = LogisticRegression(max_iter=500)
```

Train model

Then we train (fit) the classifier/model using only the features (`X_train`) and targets (`y_train`) from the train dataset ...

```
model.fit(X_train, y_train)
```

```
LogisticRegression(max_iter=500)
```

Predict

Now that model is trained, we can use it to generate predictions, using the features (`X_test`) from the test dataset ...

```
y_pred = model.predict(X_test)
```

Example: IRIS Dataset — Evaluate I

Scoring and confusion matrix

We could just compute the score using whatever metric we have picked ...

```
from sklearn.metrics import accuracy_score
accuracy_score(y_test, y_pred)
```

```
0.8333333333333334
```

But this needs context, and even if good can hide critical flaws. Lets look at the confusion matrix ...

```
from sklearn.metrics import confusion_matrix
cm = confusion_matrix(y_test,y_pred)
cm
```

```
array([[18, 1, 0],
       [ 0, 17, 6],
       [ 0,  3, 15]])
```

or, to get a nicer output, convert to a DataFrame ...

```
df_cm = pd.crosstab(target_names[y_test],target_names[y_pred])
df_cm.index.name = 'Actual'
df_cm.columns.name = 'Predicted'
df_cm
```

Predicted	setosa	versicolor	virginica
Actual			
setosa	18	1	0
versicolor	0	17	6
virginica	0	3	15

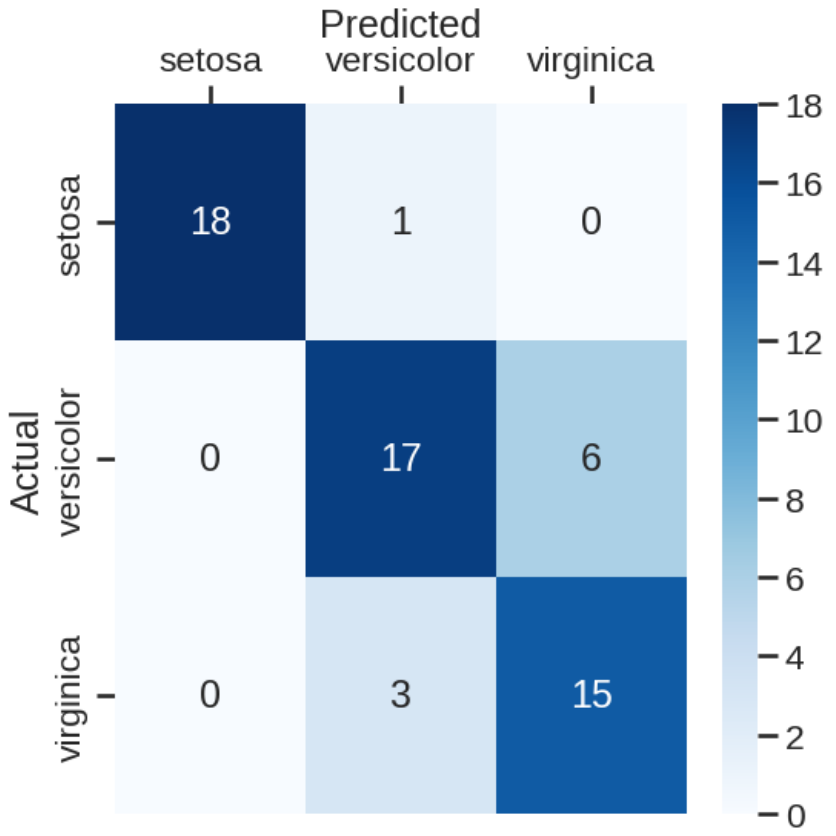
Example: IRIS Dataset — Evaluate II

The confusion matrix is fundamental in evaluating a classifier, so find a presentation/visualisation that you like and use it. Here I have a heat map representation that I tend to use.

Predicted	setosa	versicolor	virginica
Actual			
setosa	18	1	0
versicolor	0	17	6
virginica	0	3	15

The first class `setosa` was only misclassified once, while the classifier had more difficulty between the second two classes.

```
plt.figure(figsize=(6,6))
g = sns.heatmap(df_cm, annot=True, cmap="Blues")
g.xaxis.set_ticks_position("top")
g.xaxis.set_label_position('top')
```



Example: IRIS Dataset — Evaluate III

The classification report, constructed from the confusion matrix, summaries the most common metrics per class and for overall averages ...

```
from sklearn.metrics import classification_report
print(classification_report(y_test, y_pred, target_names=target_names))
```

precision (setosa) = $18/(18+0) = 1$

recall (setosa) = $18/(18+1) = 0.95$

Predicted / Actual	setosa	versicolor	virginica
setosa	18	1	0
versicolor	0	17	6
virginica	0	3	15

```
                precision    recall  f1-score   support

        setosa       1.00      0.95      0.97        19
    versicolor       0.81      0.74      0.77        23
     virginica       0.71      0.83      0.77        18

      accuracy                           0.83        60
     macro avg       0.84      0.84      0.84        60
  weighted avg       0.84      0.83      0.84        60
```

accuracy = $(18+17+15)/60 = 0.83$

f1-score (virginica) = $2/(1/0.71 + 1/0.83) = 0.77$

Motivation

Linear regression is a very powerful and flexible prediction model (Topics 7 and 8)

Can it be used to predict categorical variables, perhaps coded as numbers?

- Pros
  - Linear regression provides a principled way of combining the contributions of the predictors, whether they are numeric or not.
  - There is a lot of well-established theory and practice, e.g., in respect of collinearity.
  - The model is extremely flexible, e.g., predictors can be nonlinear functions of the features.
  - Implementations can use computing resources efficiently.
- Cons
  - Prediction is numeric value, needs to be converted to a categorical value.
  - Conversion function introduces unwelcome features, e.g., ordering and scaling, that do not apply to nominal variables.
  - Interpretation of linear regression in terms of classification performance is tricky because the conversion function needs to balance continuity against evaluating to either 0 or 1.

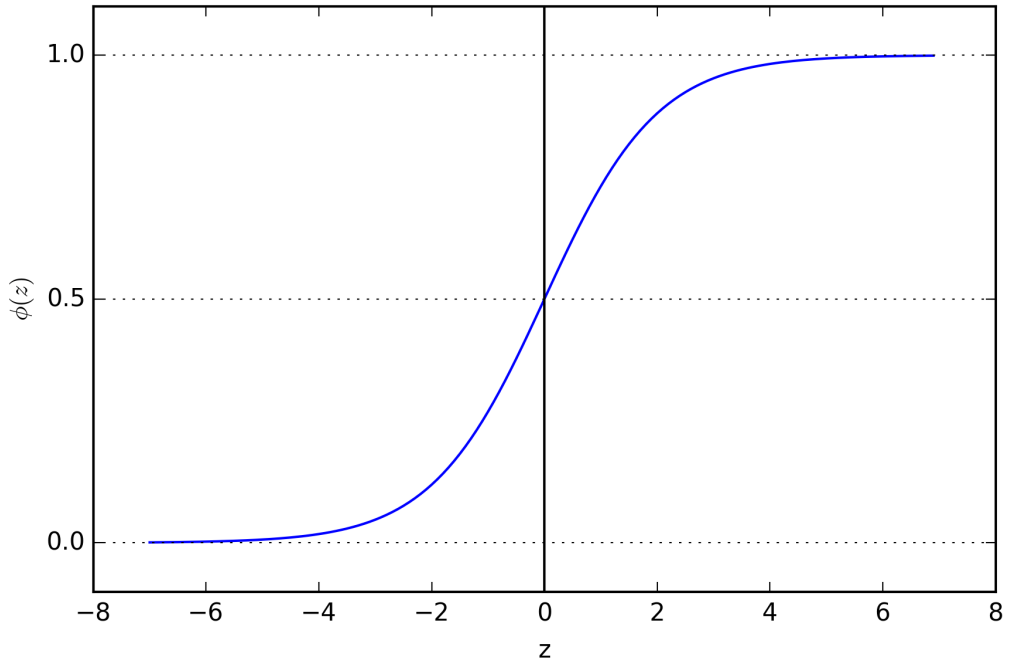
So—is there any hope for using linear regression in classification?

Probability view of Classification

When we predict a nominal value, it is equivalent to saying that the probability that a given observation takes that value is high and that it takes any other value is low.

- Probabilities are numeric, but their range is restricted to $[0, 1]$.
- We need a function of the predictors with the following properties
  - it has the range $[0, 1]$
  - it is defined over a domain which is $(-\infty, \infty)$
  - it can evaluate to 0 or 1, depending on its input.
- A line (like $y = \beta_0 + \beta_1 x$), such as we used in previous topics, does not have these properties
- Ideally, the function should be smooth and well-behaved everywhere, *and* evaluate to 0 or 1 as appropriate.
- Note that the classes are labeled as 0 or 1 (binary classification).

Introducing the logistic function



Definition 2 (logistic function)

The curve above is given by the logistic function, which can be written as $p(z) = \frac{e^z}{1+e^z}$. Letting $z = \beta_0 + \beta_1 X$, we have $p(X) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}$ for some β_0 and β_1.

Predicting the (log) odds ratio

$$
\begin{aligned}
p(X) &= \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}} && (1)\\
\implies e^{\beta_0+\beta_1 X} &= p(X) + e^{\beta_0+\beta_1 X} p(X)\\
\implies (1-p(X))e^{\beta_0+\beta_1 X} &= p(X)\\
\implies \frac{p(X)}{1-p(X)} &= e^{\beta_0+\beta_1 X}\\
\implies \log\left(\frac{p(X)}{1-p(X)}\right) &= \beta_0+\beta_1 X && (2)
\end{aligned}
$$

The expression on the left of (2) is called the *log-Odds Ratio* (or logit(X)). When $p(X) \to 1$, logit$(X) \to \infty$ and when $p(X) \to 0$, logit$(X) \to -\infty$, as required.
The expression on the right of (2) is a linear form in β, as used in linear regression.
For training: Compute logit(X) from the training labels and use linear regression to get $\{\beta_i\}$.
For prediction: Substitute X, $\{\beta_i\}$ in the logistic function (1) to obtain class round$(p(X))$.

Logistic regression summary

Training

Given (labeled) training data, convert the label to the equivalent logit value and use *maximum likelihood estimation* to look for the parameters β that make the observed data as likely as possible. Extension to multiple predictors is trivial.

Prediction

Given the fitted β, just evaluate the logistic function for a specific X. The resulting $p(X)$ will hopefully be near 0 or 1 and can be interpreted according to how "success" ($p = 1$) is defined.

Extension to categorical features

As with linear regression: create dummy (binary) indicator (0,1)-valued variables, one for each level of the categorical predictor.

Extension to non-binary targets

We can convert to extra indicator variables, but at some cost in complexity. Therefore, logistic regression is best suited to binary prediction.

Logistic regression in python

Python's `scikit-learn` and `statsmodel` libraries provide a general interface to model fitting that abstracts away logistic functions and other details.

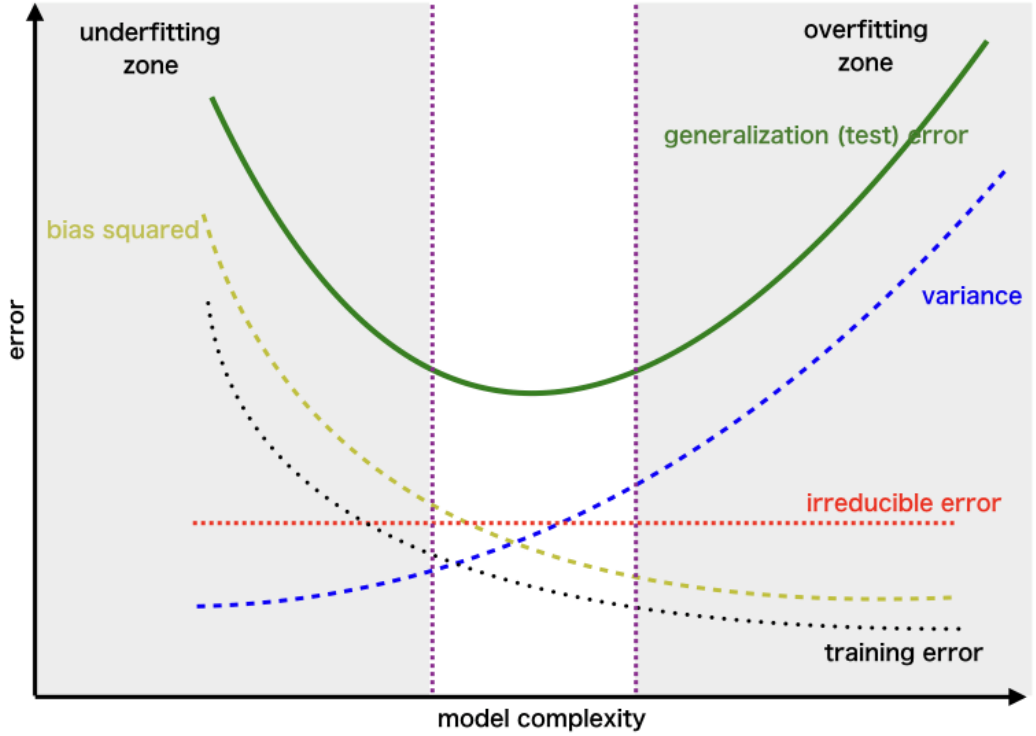
Method (Recognising the Handwritten Digits)

```
from sklearn.linear_model import LogisticRegression

# Get and configure a LogisticRegression object, with an L2 regularisation penalty
clf = LogisticRegression(penalty='l2', max_iter=7600)

# Fit the training data
clf.fit(Xtrain, ytrain)

# Using the beta parameters that have been learned and are in clf, predict (recognise) the test data
ypred = clf.predict(Xtest)
```

Review of model building in supervised learning

Goal: Choose the features and model formulation that predict the target, minimising error on the test set

- What features should we choose?
- Need enough to minimise bias due to underfitting
- . . . But not enough to increase variance due to overfitting
- So choose best subset (this is a combinatorial task (2^n))
- . . . But are the features in the dataset the best ones to choose from?
- Can we derive new ones that do better?



Features can be selected from an existing set, transformed from individual features, or derived from a set of features.

Explanatory power and number of features

- The relationship between features and targets can be very complex.
- If a feature $X[:, j]$ is very "simple", how valuable is it when it comes to explaining the target?
- If the target y is also "simple", *and* they are highly correlated, then feature $X[:, j]$ is very valuable
- Otherwise, it is less valuable, except when combined with other features, so that *the combination of features* is highly correlated with the target y.
- Example: One-hot-encoding of a categorical feature converts it into a relatively simple 0, 1-valued set of features.
- What if there was a way of converting that set of simple numerical features into a smaller set of derived numerical features *with much the same explanatory power* as the larger set?
- What if that procedure could be applied to any set of numerical features, to derive a smaller set of numerical features, *with much the same explanatory power*?

Forward selection (adding features, one by one, until the error increases) is a simple search strategy.

Feature independence in Multivariate Data

Definition 3 (Covariance)

$\sigma_{12} = E[(X_1 - \mu_1)(X_2 - \mu_2)]$. In words, for two features X_1 and X_2, with means μ_1 and μ_2, respectively, σ_{12} is a measure of the the linear dependence between them. If they are independent, we can show that $\sigma_{12} = 0$.

Definition 4 ((Variance-)Covariance Matrix)

When there are n numeric features, there are $n \times n$ pairs of covariances $\sigma_{ij}, i = 1, \ldots, n; j = 1, \ldots, n$. The resulting covariance matrix is symmetric and diagonally dominant. This matrix captures the covariance structure of the set of n features $\{X_i\}$.

We have seen the correlation matrix, which is a scaled version of the covariance matrix, with elements $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$, all the diagonal elements are 1 and the off diagonal elements satisfy $-1 < \rho_{ij} < 1$. If two features are highly correlated, adding the second into the model does not increase the explanatory power of the model. Therefore, it pays to determine the correlation matrix from the data before building any models.

Multivariate data with correlated measurements

Example 5 (Measles cases, by city, per week from 1948–1985)

This data spans the period before and after the introduction of vaccination for measles (during the mid 1960s). Measles cases are recorded per week in 7 English cities. Although the cities are not adjacent, it is likely that there will be some spatial autocorrelation. Also, by the nature of disease outbreaks, there will also be some temporal autocorrelation per city.

	Date	London	Bristol	Liverpool	Manchester	Newcastle	Birmingham	Sheffield
1	1948-01-17	240	4	51	19	52	84	11
2	1948-01-24	284	3	54	23	34	65	11
3	1948-01-31	340	5	54	31	25	106	4
4	1948-02-07	511	1	89	66	27	142	7
5	1948-02-14	649	3	73	60	47	143	3
6	1948-02-21	766	13	169	87	46	191	6
7	1948-02-28	932	5	212	61	66	208	9
8	1948-03-06	1303	4	283	79	57	290	7
9	1948-03-13	1257	15	285	56	82	310	10
10	1948-03-20	1716	9	279	85	92	425	5
11	1948-03-27	1425	3	424	63	94	481	10

Removing redundant attribues, based on correlation filters

Pearson Correlation

	London	Bristol	Liverpool	Manchester	Newcastle	Birmingham	Sheffield
London	1.000000	0.474016	0.295005	0.519947	0.520185	0.707410	0.539053
Bristol	0.474016	1.000000	0.228214	0.437572	0.374370	0.546398	0.680336
Liverpool	0.295005	0.228214	1.000000	0.431414	0.482269	0.365078	0.329118
Manchester	0.519947	0.437572	0.431414	1.000000	0.554188	0.472575	0.522391
Newcastle	0.520185	0.374370	0.482269	0.554188	1.000000	0.645766	0.535574
Birmingham	0.707410	0.546398	0.365078	0.472575	0.645766	1.000000	0.690961
Sheffield	0.539053	0.680336	0.329118	0.522391	0.535574	0.690961	1.000000

London-Birmingham has correlation *greater than* 0.7.

Spearman Correlation

	London	Bristol	Liverpool	Manchester	Newcastle	Birmingham	Sheffield
London	1.000000	0.654859	0.399211	0.589346	0.559762	0.764533	0.581148
Bristol	0.654859	1.000000	0.356830	0.598125	0.471088	0.636617	0.613336
Liverpool	0.399211	0.356830	1.000000	0.580160	0.558448	0.383332	0.421292
Manchester	0.589346	0.598125	0.580160	1.000000	0.491076	0.507557	0.577990
Newcastle	0.559762	0.471088	0.558448	0.491076	1.000000	0.591156	0.633679
Birmingham	0.764533	0.636617	0.383332	0.507557	0.591156	1.000000	0.599110
Sheffield	0.581148	0.613336	0.421292	0.577990	0.633679	0.599110	1.000000

London-Birmingham has correlation *greater than* 0.7.

Kendall Correlation

	London	Bristol	Liverpool	Manchester	Newcastle	Birmingham	Sheffield
London	1.000000	0.471882	0.268666	0.417987	0.402433	0.570474	0.416055
Bristol	0.471882	1.000000	0.243417	0.428664	0.331594	0.460080	0.449481
Liverpool	0.268666	0.243417	1.000000	0.411598	0.400088	0.260798	0.291779
Manchester	0.417987	0.428664	0.411598	1.000000	0.346396	0.354931	0.411831
Newcastle	0.402433	0.331594	0.400088	0.346396	1.000000	0.428067	0.463323
Birmingham	0.570474	0.460080	0.260798	0.354931	0.428067	1.000000	0.432066
Sheffield	0.416055	0.449481	0.291779	0.411831	0.463323	0.432066	1.000000

London-Birmingham has correlation *less than* 0.7.

Observations

- Critical level of correlation $\rho^{(\text{crit})} = 0.7$, so one of London or Birmingham can be dropped.
- The Spearman correlations are particularly high, so more correlation might be present.
- The Kendall correlations are inconclusive.

Working with high-dimensional data

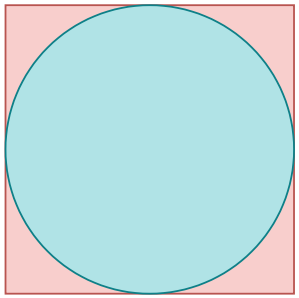
Definition 6 (Curse of Dimensionality)

High dimensions do not just require more computing resources and make interpretation more difficult. They also make it more difficult to capture data that samples very high dimensional spaces efficiently. As the dimension d increases, most of the volume of a hypercube is near the corners, not near the centre, where data might be easiest to collect. This makes estimating parameters much more difficult.

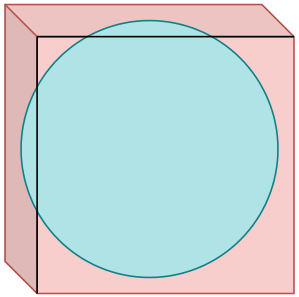
- The proof is based on the fact that, as the dimension d tends to infinity, the *ratio* of the volume of the maximum hypersphere inscribed inside the hypercube of the same dimension, tends to 0. Thus there are good reasons to prefer low dimension approximations to high dimensional space.
- In 2D: imagine the largest circle fitting inside a square; ratio is $\frac{\pi r^2}{4r^2} = \frac{\pi}{4} \approx 0.79$.
- In 3D: imagine the largest sphere fitting inside a cube; ratio is $\frac{(4/3)\pi r^3}{8r^3} = \frac{\pi}{6} \approx 0.52$.
- More generally

$$\frac{V_{\text{hypersphere}}}{V_{\text{hypercube}}} = \frac{\pi^{d/2}}{d2^{d-1}\Gamma(d/2)} \to 0 \text{ when } d \to \infty$$

Impact: Harder to collect data that samples high-dimensional space (which is mostly in the "corners"...), so harder to estimate such models. Analogy: "corner cases" when testing software.



2D: circle in square



3D: sphere in cube

Feature reduction

- Sometimes it is possible to use intuition to reduce the dimension, by omitting selected attributes.
- Another possibility is to look for groups of correlated attributes (c.f., *mediation*), such as the London and Birmingham measles cases above, and just choose 1 of these.
- More generally, there are techniques that search for a subspace with specified dimension d' of the attributes that captures most of the variance of the full set of attributes having dimension d, where $d' < d$ (often $d' \ll d$).
- The best known of these techniques is *Principal Components Analysis* (PCA).
- Refer to worked examples of PCA in the lab notebook.

Review of Classification 1

- Classification is one of the most common machine learning tasks
- For regression, the focus is on residuals, for classification it is on the confusion matrix
- Performance metrics are based on ratios and independent of the algorithm used
- Trade-offs are needed: Type 1 vs Type 2 errors and associated classification metrics
- Builds upon all the existing EDA, model building and multivariate analysis we saw before

In classification 2, we introduce 2 new classification algorithms